Class XII Session 2025-26
Subject - Applied Mathematics
Sample Question Paper - 10

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are case study-based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and one sub-part each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. If A and B are matrices of the same order, then $AB^T = B^TA$ is a: **[1]**

a) unit matrix   b) skew-symmetric matrix

c) null matrix   d) symmetric matrix

2. A specific characteristic of a population is known as a **[1]**

a) mean   b) statistic

c) a sample   d) parameter

3. The present value of a perpetuity of ₹ R payable at the end of each payment period, when the money is worth i per period, is given by: **[1]**

a) Ri   b) R - Ri

c) $\frac{R}{i}$ d) R + $\frac{R}{i}$

4. The maximum value of the function z = 7x + 5y, subject to constraints $x \leq 3, y \leq 2, x \geq 0, y \geq 0,$ is: **[1]**

a) 31   b) 37

c) 10   d) 21

[1]

5. If A = $\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and A = A^T, then:

a) x = y

b) x + y = 5

c) x = 0, y = 5

d) x - y = 5

6. The probability distribution of a discrete random variable X is given below: [1]

X	2	3	4	5
P(X)	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

The value of E(X) is:

a) 16

b) 48

c) 8

d) 32

7. Let X be a discrete random variable. Then the variance of X is: [1]

a) $E(X^2) + (E(X))^2$

b) $E(X^2)$

c) $E(X^2) - (E(X))^2$

d) $\sqrt{E(X^2) - (E(X))^2}$

8. $y = ae^{mx} + be^{-mx}$, where a, b are arbitrary constants satisfies which of the following differential equation? [1]

a) $\frac{dy}{dx}$ + my = 0

b) $\frac{d^2y}{dx^2}$ + m^2y = 0

c) $\frac{dy}{dx}$ - my = 0

d) $\frac{d^2y}{dx^2}$ - m^2y = 0

9. A boat covers 8 km in one hour along the stream and 2 km in one hour against the stream. The speed of the stream in km/hr is [1]

a) 3

b) 5

c) 2

d) 4

10. If A is a square matrix of order 3 such that A(adj A) = $\begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then |adj A| is equal to [1]

a) 4

b) -2

c) -8

d) -4

11. The ratio in which a grocer mixes two varieties of pulses costing ₹ 85 per kg and ₹ 100 per kg respectively so as to get a mixture worth 92 per kg, is: [1]

a) 7 : 5

b) 8 : 7

c) 7 : 8

d) 5 : 7

12. Given that x, y and b are real numbers and x $\geq$ y, b > 0, then [1]

a) $\frac{x}{b} < \frac{y}{b}$

b) $\frac{x}{b} > \frac{y}{b}$

c) $\frac{x}{b} \geq \frac{y}{b}$

d) $\frac{x}{b} \leq \frac{y}{b}$

13. A boat goes downstream at u km/hr and upstream at v km/hr. The speed of the stream, in km/hr, is [1]

a) u + v

b) u - v

c) $\frac{1}{2}$(u + v)

d) $\frac{1}{2}$(u - v)

14. For the constraint of a linear optimizing function $z = x_1 + x_2$, given by $x_1 + x_2 < 1$, $3x_1 + x_2 > 3$ and $x_1, x_2 > 0$ **[1]**

a) There is no feasible region
b) There are two feasible regions
c) There are infinite feasible regions
d) There are finite feasible regions

15. In an L.P.P. if the objective function Z = ax + by has same maximum value on two corner points of the feasible region, then the number of points at which the maximum value of Z occurs is **[1]**

a) infinite
b) 2
c) finite
d) 0

16. The assumed hypothesis which is tested for rejection considering it to be true is called **[1]**

a) true hypothesis
b) null hypothesis
c) alternative hypothesis
d) simple hypothesis

17. If the demand function for a commodity is $p = 20 - 2x - x^2$ and the market demand is 3 units, then consumer's surplus is **[1]**

a) 27
b) 47
c) 42
d) 38

18. The graph of time series is called: **[1]**

a) Historigram
b) Straight line
c) Ogive
d) Histogram

19. Suppose A, B and C are three square matrices of the same order such that A B is invertible. **[1]**

Assertion (A): If AB = AC, then B = C

Reason (R): A is invertible.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** The absolute maximum value of the function $2x^3 - 24x$ in the interval [1, 3] is 89. **[1]**

Reason (R): The absolute maximum value of the function can be obtained from the value of the function at critical points and at boundary points.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Mr Tripathi invested ₹ 10000 in a company's fund. His yearly investment values are shown in the table given below: **[2]**

Year	0	1	2	3
Amount (in ₹)	10000	13000	11000	9400

Calculate CAGR of his investment.

OR

Arnav owns a produce truck, invested ₹700 in purchasing the truck, some other initial admin related and insurance expenses of ₹1500 to get the business going, and has now a day to day expense of ₹500/p.m. Consider hypothetically that her everyday profit is ₹550/p.m. (ideally, it will be based on sales). At the end of 6 months, Anna takes up her accounts and calculates her rate of return.

22. The following table shows the annual rainfall (in mm) recorded for Cherrapunji, Meghalaya: **[2]**

Year	**Rainfall (in mm)**
2001	1.2
2002	1.9
2003	2
2004	1.4
2005	2.1
2006	1.3
2007	1.8
2008	1.1
2009	1.3

Determine the trend of rainfall by 3-year moving average.

23. Evaluate: $\int\limits_{1}^{2} \frac{3x}{9x^2-1} dx$ **[2]**

24. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, find the values of a and b. **[2]**

OR

Find matrices X and Y, if X + Y = $\begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and X - Y = $\begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

25. In what ratio must a person mix two sugar solutions of 30% and 50% concentration respectively so as to get a solution of 45% concentration? **[2]**

Section C

26. Madhu exchanged her old car valued at ₹ 1,50,000 with a new one priced at ₹ 6,50,000. She paid ₹ x as down payment and the balance in 20 monthly equal instalments of ₹ 21,000 each. The rate of interest offered to her is 9% p.a. Find the value of x. [Given that: $(1.0075)^{-20}$ = 0.86118985] **[3]**

27. Determine the order and the degree (when defined) differential equations: **[3]**

$$5\frac{d^2y}{dx^2} = \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{1}{4}}$$

OR

Solve the initial value problem: $x\frac{dy}{dx}$ + y = x log x, y(1) = $\frac{1}{4}$

28. The demand function for a commodity is p = 20 $e^{-x/10}$. Find the consumer's surplus at equilibrium price p = 2. (Given $\log_{10} e$ = 0.4343) **[3]**

29. The average number, in lakhs, of working days lost in strikes during each year of the period (1981 - 90) was as under: **[3]**

1981	1982	1983	1984	1985	1986	1987	1988	1989	1990

1.5	1.8	1.9	2.2	2.6	3.7	2.2	6.4	3.6	5.4

Calculate the three-yearly moving average and draw the moving average graph.

30. Ten students are selected at random from a college and their heights are found to be 100, 104, 108, 110, 118, 120, 122, 124, 126 and 128 cms. In the light of these data, discuss the suggestion that the mean height of the students of the college is 110 cms. (Given $t_9(0.05) = 2.262$) **[3]**

31. From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the items in the sample are drawn one by one without replacement, find: **[3]**
    i. The probability distribution of X
    ii. Mean of X
    iii. Variance of X

OR

Two bad eggs are accidentally mixed up with ten good ones. Three eggs are drawn at random with replacement from this lot. Compute the mean for the number of bad eggs drawn.

Section D

32. A company sells two different products, A and B. The two products are produced in a common production process, which has a total capacity of 500 man-hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The market has been surveyed and company officials feel that the maximum number of units of A that can be sold is 70 and that for B is 125. If the profit is ₹20 per unit for product A and ₹15 per unit for product B, how many units of each product should be sold to maximize profit? **[5]**

OR

A dealer in rural area wishes to purchase a number of sewing machines. He has only ₹ 57600 to invest and has space for atmost 20 items. An electronic sewing machine costs him ₹ 3600 and a manually operated sewing machine ₹ 2400. He can sell an electronic sewing machine at a profit of ₹ 220 and a manually operated sewing machine at a profit of ₹180. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit. Make it as an L.P.P. and solve it graphically.

33. Find the probability distribution of the number of green balls drawn when 3 balls are awn, one by one, without replacement from a bag containing 3 green and 5 white balls. **[5]**

OR

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of X. Find the mean and the variance of this distribution.

34. Two vessels P and Q contain milk and water in the ratio 5 : 3 and 13 : 3 respectively. In what ratio mixtures from two vessels should be mixed to get a new mixture containing milk and water in the ratio 3 : 1 respectively? **[5]**

35. A person amortizes a loan of ₹ 1500000 for renovation of his house by 8 years mortgage at the rate of 12% p.a. compounded monthly. Find **[5]**
    i. the equated monthly installment
    ii. the principal outstanding at the beginning of 40th month.
    iii. the interest paid in 40^{th} payment.

    [Given $(1.01)^{96} = 2.5993$, $(1.01)^{57} = 1.7633$]

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

A firm has the cost function $C(x) = \frac{x^3}{3} - 7x^2 + 111x + 50$ and demand function x = 100 - p.

i. Find the total revenue function. (1)
ii. Find the total profit function. (1)
iii. Find the value of x for which profit is maximum. (2)

OR

Find the marginal revenue when x = 10 units. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

What Is a Sinking Fund?

A sinking fund contains money set aside or saved to pay off a debt or bond. A company that issues debt will need to pay that debt off in the future, and the sinking fund helps to soften the hardship of a large outlay of revenue. A sinking fund allows companies that have floated debt in the form of bonds gradually save money and avoid a large lump-sum payment at maturity.

Example:

- Cost of Machine: ₹2,00,000/-
- Effective Life: 7 Years
- Scrap Value: ₹30,000/-
- Sinking Fund Earning Rate: 5%
- The Expected Cost of New Machine: ₹3,00,000/-

i. What is the money required for a new machine after 7 years? (1)
ii. What is the value of A, i and n here? (1)
iii. What formula will you use to get the requisite amount? (2)

OR

What amount should the company put into a sinking fund earning 5% per annum to replace the machine after its useful life? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

On his birthday, Ishan decided to donate some money to children of an orphanage home. If there were 8 children less, every one would have got ₹ 10 more. However, if there were 16 children more, every one would have got ₹ 10 less.

Let the number of children be x and amount distributed by lshan to each child be ₹ y.

i. Write matrix equation represent the information given above. (1)
ii. If A is the coefficient matrix of above situation, then what is (adj A)? (1)
iii. Find the number of children and amount donated by Ishan. (2)

OR

If the coefficient matrix A satisfy the following matrix equation $A^2 - kA + 20I = 0$, then find k. (2)

Solution

Section A

1.

(b) skew-symmetric matrix

Explanation:

skew-symmetric matrix

2.

(d) parameter

Explanation:

parameter

3.

(c) $\frac{R}{i}$

Explanation:

$\frac{R}{i}$

4. **(a)** 31

Explanation:

31

5. **(a)** x = y

Explanation:

$A = A^T$

$\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix} = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix}$

x = y

6.

(d) 32

Explanation:

We know that, $\sum P(X) = 1$

$\Rightarrow \frac{5}{k} + \frac{7}{k} + \frac{9}{k} + \frac{11}{k} = 1$

$\Rightarrow \frac{32}{k} = 1$

$\therefore k = 32$

7.

(c) $E(X^2) - (E(X))^2$

Explanation:

Since, the variance of a discrete random variable X is given by:

$Var(X) = E(X^2) - (E(X))^2$

8.

(d) $\frac{d^2y}{dx^2} - m^2y = 0$

Explanation:

$y = ae^{mx} + be^{-mx}$

$\Rightarrow \frac{dy}{dx} = mae^{mx} - bme^{-mx}$

and $\frac{d^2y}{dx^2}$ = m^2 ae^{mx} + m^2be^{-mx} = $m^2(ae^{mx} + be^{-mx})$

$\Rightarrow \frac{d^2y}{dx^2}$ = m^2y i.e. $\frac{d^2y}{dx^2}$ - m^2y = 0

9. **(a)** 3

Explanation:

Let the boat and strem speed be x and y km/hr

Speed in downstream = (x + y) km/hr

Speed in upstream = (x - y) km/hr

$t_{downstream} = \frac{8}{x+y}$

$1 = \frac{8}{x+y}$

x + y = 8 ...(i)

$t_{upstream} = \frac{2}{x-y}$

$1 = \frac{2}{x-y}$

x - y = 2 ...(iii)

(i) + (ii)

2x = 10

x = 5 km/hr

Put x = 5 in equation (i)

x = y = 8

5 + y = 8

y = 3 km/hr

Hence, speed of stream = 3 km/hr.

10. **(a)** 4

Explanation:

A(adj A) = $-2I_3 \Rightarrow |A| = -2$

So, |adj A| = $|A|^2 = (-2)^2 = 4$

11.

(b) 8 : 7

Explanation:

8 : 7

12.

(c) $\frac{x}{b} \geq \frac{y}{b}$

Explanation:

$\frac{x}{b} \geq \frac{y}{b}$

13.

(d) $\frac{1}{2}$(u - v)

Explanation:

Let the boat and stream speed be x km/hr and y km/hr

downstream speed = x + y

u = x + y ...(i)

upstream speed = x - y

v = x - y ...(ii)

(i) - (ii)

2y = u - v

$y = \frac{u-v}{2}$

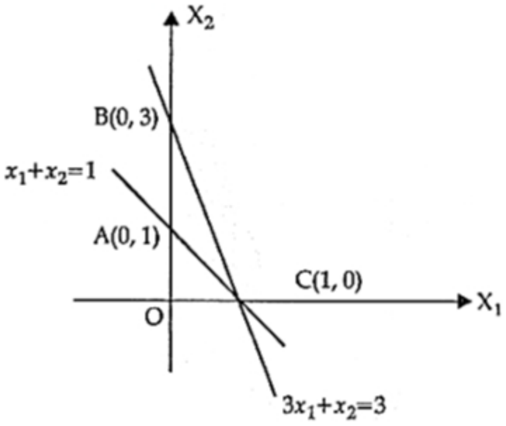
$y = \frac{1}{2}$(u - v)

$\therefore$ speed of stream = $\frac{1}{2}$(u - v)

14. **(a)** There is no feasible region

Explanation:

Clearly from graph there is no 1 feasible region.



15. **(a)** infinite

Explanation:

Since Z = ax + by has maximum value at two comer points. So, Z has the same maximum value at every point of the line segment joining these two points. Hence, maximum value of Z occurs at infinite points.

16.

(b) null hypothesis

Explanation:

null hypothesis

17. **(a)** 27

Explanation:

Given $p = 20 - 2x - x^2$ and $x_0 = 3$

So, $p_0 = 20 - 2 \times 3 - 32 \Rightarrow p_0 = 5$

$CS = \int_0^3 \left(20 - 2x - x^2\right) dx - 3 \times 5$

$= \left[20x - x^2 - \frac{x^3}{3}\right]_0^3 - 15 = (60 - 9 - 9) - 15 = 27$

18.

(b) Straight line

Explanation:

Straight line

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Given AB is invertible $\Rightarrow |AB| \neq 0$

$\Rightarrow |A| \cdot |B| \neq 0 \Rightarrow |A| \neq 0$

$\Rightarrow$ A is invertible $\Rightarrow$ Reason is true.

Now, AB = AC (given)

Pre-multiplying by A^{-1} both the sides ($\because |A| \neq 0 \Rightarrow A^{-1}$ exists)

$A^{-1}(AB) = A^{-1}(AC)$

$\Rightarrow (A^{-1} A) B = (A^{-1} A)C \Rightarrow IB = IC$

$\Rightarrow B = C$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

20.

(d) A is false but R is true.

Explanation:

Let $f(x) = 2x^3 - 24x$

$\Rightarrow f'(x) = 6x^2 - 24 = 6(x^2 - 4)$

= 6(x + 2) (x - 2)
For maxima or minima put f'(x) = 0.
$\Rightarrow$ 6(x + 2)(x - 2) = 0
$\Rightarrow$ x = 2, -2
We first consider the interval [1, 3].
So, we have to evaluate the value of f at the critical point x = 2 $\in$ [1, 3] and at the end points of [1, 3].
At x = 1, f(1) = $2 \times 1^3 - 24 \times 1 = -22$
At x = 2, f(2) = $2 \times 2^3 - 24 \times 2 = -32$
At x = 3, f(3) = $2 \times 3^3 - 24 \times 3 = -18$
$\therefore$ The absolute maximum value of f(x) in the interval [1, 3] is -18 occurring at x = 3.
Hence, Assertion is false and Reason is true.

Section B

21. Given P.V. = ₹ 10000, F.V. = ₹ 9400, n = 3 years

So, CAGR = $\left(\frac{9400}{10000}\right)^{\frac{1}{3}} - 1 = (0.94)^{\frac{1}{3}} - 1$

Let x = $(0.94)^{\frac{1}{3}}$

$\Rightarrow$ log x = $\frac{1}{3}$ log 0.94 = $\frac{1}{3} \times \bar{1}.9731$

= $\frac{1}{3}$ (-0.0269) = -0.00897

$\Rightarrow$ log x = $\bar{1}.99103$

$\Rightarrow$ x = antilog $\bar{1}.99103$ = 0.9795

So, CAGR = 0.9795 - 1 = -0.0205

Hence, CAGR = -0.0205 $\times$ 100 % = -2.05 %.

OR

Total Initial Investment: ₹700 + ₹1,500 = ₹2,200
Everyday Expenses = ₹500
Total Expenses for 6 months = ₹3,000
Everyday Returns = ₹550
Total Returns for 6 months = ₹3,300 So, Rate of Return

$= \left(\frac{\text{Total Returns - Total Expenses}}{\text{Total Initial Investment}}\right) \times 100$

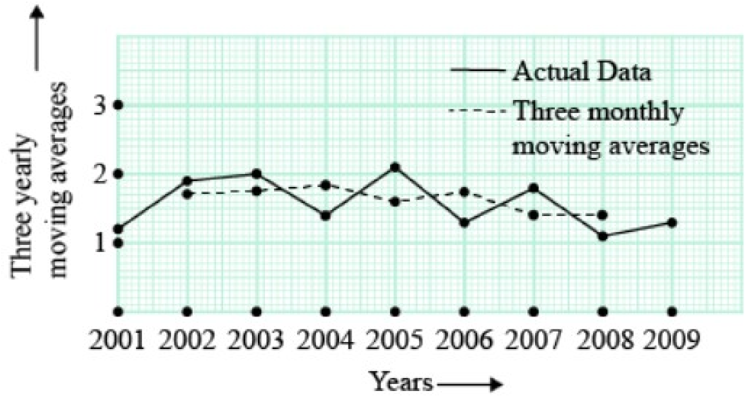
$= \left(\frac{₹3{,}300 - ₹3{,}000}{₹2{,}200}\right) \times 100$

= 13.63%

22.

Year	Rainfall (in mm)	Three Yearly Moving total	Three yearly Moving Average
2001	1.2		
2002	1.9	5.1	1.7
2003	2	5.3	1.76
2004	1,4	5.5	1.83
2005	2.1	4.8	1.6
2006	1.3	5.2	1.73
2007	1.8	4.2	1.4
2008	1.1	4.2	1.4
2009	1.3		

The points are joined by a line segment to obtain the graph to understand the trend.



23. Put $9x^2 - 1 = t \Rightarrow 18x\,dx = dt \Rightarrow 3x\,dx = \frac{1}{6}dt$.

When x = 1, $t = 9.1^2 - 1 = 8$ and when x = 2, $t = 9.2^2 - 1 = 35$.

$\therefore \mathrm{I} = \frac{1}{6}\int_8^{35} \frac{1}{t}dt = \frac{1}{6}[\log|t|]_8^{35} = \frac{1}{6}(\log 35 - \log 8) = \frac{1}{6}\log\frac{35}{8}$.

24. The corresponding elements of two equal matrices are equal.

$\therefore \begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

$\Rightarrow a + b = 6$ and $ab = 8$

$\Rightarrow a + \frac{8}{a} = 6$ [$\because ab = 8 \Rightarrow b = \frac{8}{a}$]

$\Rightarrow a^2 + 8 = 6a \Rightarrow a^2 - 6a + 8 = 0 \Rightarrow (a - 4)(a - 2) \Rightarrow a = 2, 4$

Now, a = 2 and ab = 8 $\Rightarrow$ b = 4

and, a = 4 and ab = 8 $\Rightarrow$ b = 2

Hence, a = 2 and b = 4, or a = 4 and b = 2

OR

$x + y + x - y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

$2x = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$

$x = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$

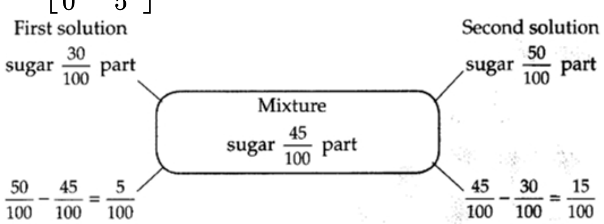
and

$(x + y) - (x - y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

$2y = \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$

$y = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$

25.



$\therefore \frac{\text{Quantity of first solution}}{\text{Quantity of second solution}} = \frac{\frac{5}{100}}{\frac{15}{100}} = \frac{1}{3}$ i.e. 1 : 3

Section C

26. Madhu paid the balance in 20 monthly installments of ₹ 21000 each

Let Principle = P, $i = \frac{9}{1200} = 0.0075$, n = 20 and E = 21000

$E = \frac{Pi}{1-(1+i)^{-n}}$

$\Rightarrow 21000 = \frac{P\times(0.0075)}{1-(1.0075)^{-20}}$

$\Rightarrow 21000 = \frac{P\times(0.0075)}{1-0.8611}$

$\Rightarrow 21000 = \frac{P\times(0.0075)}{0.1389}$

$\Rightarrow 21000 \times 0.1389 = P \times (0.0075)$

$\Rightarrow$ P = 388920

Thus, the balance is ₹ 388920

Madhu exchanged her old car valued at ₹ 150000 and a new one priced at ₹ 650000.

So, Madhu had ₹ 500000 after the exchange.

She paid approximately ₹ 388920in the form of monthly installments.

Therefore, the down payment x = 500000 - 388920 = 111080.

Hence, the value of x is 111080.

27. The given differential equation can be written as

$625\left(\frac{d^2y}{dx^2}\right)^4 = 1 + \left(\frac{dy}{dx}\right)^2$

It is of order 2 and degree 4.

OR

We have, x$\frac{dy}{dx}$ + y = x log x

$\Rightarrow \frac{dy}{dx} + \frac{1}{x}y$ = log x ...(i)

This is linear differential equation of the form $\frac{dy}{dx}$ + Py = Q with P = $\frac{1}{x}$ and Q = log x

$\therefore$ I.F. = $e^{\int \frac{1}{x}dx}$ = $e^{\log x}$ = x [$\because$ x > 0]

Multiplying both sides of (i) by I.F. = x, we get

x$\frac{dy}{dx}$ + y = x log x

Integrating with respect to x, we ge

yx = $\int \underset{II}{x} \underset{I}{\log x}\, dx$ [Using: y (I.F.) = $\int$ Q (I.F.) dx + C]

$\Rightarrow$ yx = $\frac{x^2}{2}$(log x)$\frac{1}{2}\int$ x dx

$\Rightarrow$ yx = $\frac{x^2}{2}$(log x) - $\frac{x^2}{4}$ + C ...(ii)

It is given that y (1) = $\frac{1}{4}$ i.e. y = $\frac{1}{4}$ where x = 1. Putting x = 1 and y = $\frac{1}{4}$ in (ii), we get

$\frac{1}{4}$ = 0 -$\frac{1}{4}$ + C $\Rightarrow$ C = $\frac{1}{2}$

Putting C = $\frac{1}{2}$ in (ii), we get

xy = $\frac{x^2}{2}$(log x) -$\frac{x^2}{2}$ + $\frac{1}{2}$ $\Rightarrow$ y = $\frac{1}{2}$x log x - $\frac{x}{4} + \frac{1}{2x}$

Hence, y = $\frac{1}{2}$ x log x - $\frac{x}{4} + \frac{1}{2x}$ is the solution of the given differential equation.

28. Given, the demand function is

p = $20e^{-x/10}$ ...(i)

and the equilibrium price p_0 = 2.

Substituting this value of p_0 = 2 in (i), we get

$2 = 20e^{-x_0/10} \Rightarrow e^{-x_0/10} = \frac{1}{10}$ ...(ii)

$\Rightarrow e^{x_0/10} = 10 \Rightarrow \log_e 10 = \frac{x_0}{10}$

$\Rightarrow$ x_0 = 10 $\log_e$ 10 $= \frac{10}{\log_{10} e} = \frac{10}{0.4343} = \frac{100000}{4343}$

$\Rightarrow$ x_0 = 23.03 ...(iii)

$\therefore$ CS $= \int_0^{x_0} 20e^{-x/10}dx - x_0 \times p_0$

$= 20\left[\frac{e^{-x/10}}{-\frac{1}{10}}\right]_0^{x_0} - 23.03 \times 2$ (using (iii))

= -200[$\left[e^{-x_0/10}\right.$ - e^0] - 46.06

$= -200\left[\frac{1}{10} - 1\right] - 46.06$ (using (ii))

= 180 - 46.06 = 133.94

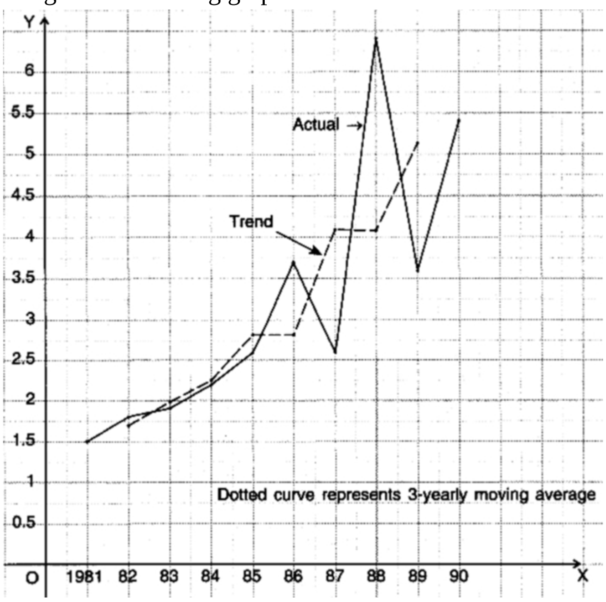
Hence, consumer's surplus is 133.94

29. First three-yearly moving average = $\frac{1.5+1.8+1.9}{3} = \frac{5.2}{3}$ = 1.73 and is placed against 1982; second three-yearly moving average = $\frac{1.8+1.9+2.2}{3} = \frac{5.9}{3}$ = 1.97 and is placed against 1983, and so on.

Calculation of 3-year moving average:

Year	Days lost (in lakhs)	3-yearly moving total	3-yearly moving average
1981	1.5	1/3	
1982	1.8	5.2	1.73
1983	1.9	5.9	1.97
1984	2.2	6.7	2.23
1985	2.6	8.5	2.83
1986	3.7	8.5	2.83
1987	2.2	12.3	4.1
1988	6.4	12.2	4.07
1989	3.6	15.4	5.13
1990	5.4		

We get the following graph from the above data:



30. We define

Null Hypothesis H_0: There is no significant difference between the sample mean and hypothetical population mean 110 cm.

Alternate hypothesis H_1: The sample mean is not same as the population mean.

Let the sample statistic t be given by

$$t = \frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}}$$

Let us now compute the sample mean ($\bar{X}$) and S.

Computation of $\bar{X}$ and S

x_i	$x_i - \bar{X}$	$(x_i - \bar{X})^2$
100	-16	256
104	-12	144
108	-8	64
110	-6	36
118	2	4
120	4	16
122	6	36
124	8	64
126	10	100
128	12	144
$\sum x_i = 1160$		$\sum_{i=1}^{10} (x_i - \bar{X})^2 = 864$

$\therefore t = \frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}}$

$\Rightarrow t = \frac{116-110}{9.798} \times \sqrt{10} = \frac{6}{9.798} \times$ 3.162 = 1.94 [$\because \mu$ = 110]

We find that $\sum_{i=1}^{10} x_i = 1160$, and n = 10

$\therefore \bar{X} = \frac{1}{n}\sum_{i=1}^{10} x_i = \frac{1160}{10}$ = 116

From the table, we find that $\sum_{i=1}^{10}\left(x_i - \bar{X}\right)^2$ = 864

$\therefore S^2 = \frac{1}{n-1}\sum_{i=1}^{10}\left(x_i - \bar{X}\right)^2 \Rightarrow S^2 = \frac{1}{9} \times 864 \Rightarrow S = \frac{\sqrt{864}}{3} = \frac{29.393}{3} = 9.798$

The sample statistic follows student's t-distribution with v = (10 - 1) = 9 degrees of freedom. We shall now compare this calculated value with the tabulated value of t for 9 degrees of freedom at a certain level of significance. It is given that t_9 (0.05) = 2.262

$\therefore$ Calculated |t| = 1.94 < 2.262 = t_9 (0.05)

i.e. Calculated |t| < tabulated t_9 (0.05)

So, we accept the null hypothesis. Hence, the sample mean is same as the population means.

Consequently, the mean height of the students of the college is 110 cm.

31. It is clear that X can assume values 0, 1, 2, 3 such that,

$P(X = 0) = \frac{^7C_4}{^{10}C_4} = \frac{1}{6}$, $P(X = 1) = \frac{^3C_1 \times {}^7C_3}{^{10}C_4} = \frac{1}{2}$

$P(X = 2) = \frac{^3C_2 \times {}^7C_2}{^{10}C_4} = \frac{3}{10}$, and $P(X = 3) = \frac{^3C_3 \times {}^7C_1}{^{10}C_4} = \frac{1}{30}$

Therefore, the probability distribution of X is as follows:

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

Computation of mean and variance :

x_i	$P(X = x_i) = p_i$	$p_i x_i$	$p_i x_i^2$
0	$\frac{1}{6}$	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{3}{10}$	$\frac{3}{5}$	$\frac{6}{5}$
3	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{3}{10}$
		$\Sigma p_i x_i = \frac{12}{10}$	$\Sigma p_i x_i^2 = 2$

Thus, we have $\Sigma p_i x_i = \frac{12}{10} = \frac{6}{5}$ and $\Sigma p_i x_i^2 = 2$

$\therefore \bar{X}$ = Mean = $\Sigma p_i x_i = \frac{6}{5}$

and, Var(X) = $\Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = 2 - \left(\frac{6}{5}\right)^2 = 2 - \frac{36}{25} = \frac{14}{25}$

Hence, Mean = $\frac{6}{5}$ and Variance = $\frac{14}{25}$

OR

Let X be a random variable denoting the number of bad eggs in a sample of 3 eggs drawn from a lot containing 2 bad eggs and 10 good eggs. Then, X can take the values 0,1 and 2.

Now,we have,

P(X = 0) = P(no bad egg)

$= \frac{^{10}C_3}{^{12}C_3} = \frac{120}{220} = \frac{12}{22}$

P(X = 1) = P(1 bad egg)

$= \frac{^2C_1 \times {}^{10}C_2}{^{12}C_3} = \frac{90}{220} = \frac{9}{22}$

P(X = 2) = P(2 bad eggs)

$= \frac{^2C_2 \times ^{10}C_1}{^{12}C_3} = \frac{10}{220} = \frac{1}{22}$

Thus, the probability distribution of X is given by

X	0	1	2
P(X)	$\frac{12}{22}$	$\frac{9}{22}$	$\frac{1}{22}$

Computation of mean :

x_i	p_i	$p_i x_i$
0	$\frac{12}{22}$	0
1	$\frac{9}{22}$	$\frac{9}{22}$
2	$\frac{1}{22}$	$\frac{1}{11}$
		$\sum p_i x_i = \frac{1}{2}$

Therefore, Mean= $\sum p_i x_i = 0 + \frac{9}{22} + \frac{1}{11} = \frac{11}{22} = \frac{1}{2}$

Section D

32. Let x units of product A and y units of product B were manufactured.

Clearly, $x \geq 0, y \geq 0$

It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The two products are produced in a common production process, which has a total capacity of 500 man-hours.

$5x + 3y \leq 500$

The maximum number of units of A that can be sold is 70 and that for B is 125.

$x \leq 70$

$y \leq 125$

If the profit is ₹20 per unit for product A and ₹15 per unit for product B.

Therefore, profit x units of product A and y units of product B is ₹20x and ₹15y respectively.

Total profit = Z = 20x + 15y

The mathematical formulation of the given problem is

Max Z = 20x + 15y

subject to

$5x + 3y \leq 500$

$x \leq 70$

$y \leq 125$

$x \leq 0, y \leq 0$

First we will convert inequations into equations as follows:

5x + 3y = 500, x = 70, y = 125, x = 0 and y = 0

Region represented by $5x + 3y \leq 500$:

The line 5x + 3y = 500 meets the coordinate axed at A_1 (100, 0) and B_1 (0, $\frac{500}{3}$) respectively. By joining these points we obtain the line 5x + 3y = 500. Clearly (0, 0) satisfies the 5x + 3y = 500. So, the region which contains the origin represents the solution set of the inequation $5x + 3y \leq 500$

Region represented by $x \leq 70$:

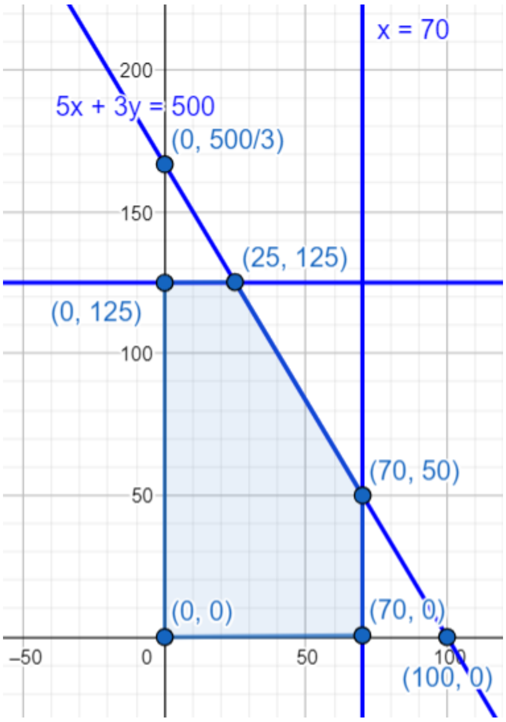
The line x = 70 is the line that passes through C_1 (70, 0) and is parallel to Y-axis. The region to the left of line x = 70 will satisfy the inequation $x \leq 70$. The region represented by $y \leq 125$:

The line y = 125 is the line that passes through D_1 (0, 125) and is parallel to X-axis. The region below the line y = 125 will satisfy the inequation $y \leq 125$.

The region represented by $x \geq 0$ and $y \geq 0$:

Since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x \geq 0$, and $y \geq 0$.

The feasible region determined by the system of constraints $5x + 3y \leq 500$, $x \leq 70$, $y \leq 125$, $x \geq 0$ and $y \geq 0$ are as follows.



The corner points are O(0, 0), D_1 (0, 125), E_1 (25, 125), F_1 (70, 50) and C_1 (70, 0). The values of Z at the corner points are

Corner Points	Z = 20x + 15y
O	0
D_1	1875
E_1	2375
F_1	2150
C_1	1400

The maximum value of Z is 2375 which is at E_1 (25, 125)

Thus, the maximum profit is ₹2375, 25 units of A and 125 units of B should be manufactured.

OR

Let x and y be the number of electronic and manually operated sewing machines that a dealer purchases and sells, then the problem can be formulated as an L.P.P.as follows:

Maximize the profit (in ₹) Z = 220x + 180y subject to the constraints"

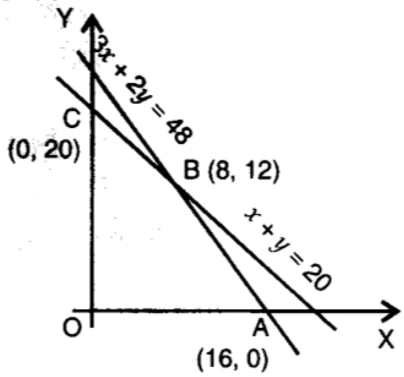
$3600x + 2400y \leq 57600$ (investment constraint)

i.e. $3x + 2y \leq 48$

$x + y \leq 20$ (storage constraint)

$x \geq 0, y \geq 0$ (non-negativity constraints)

Draw the lines 3x + 2y = 48 and x + y = 20, and shade the region satisfied by the above inequalities. The feasible region is the polygon OABC, which is convex and bounded.



The corner points are O(0, 0), A(16, 0), B(8,12) and C(0, 20).

The values of Z (in ₹) = 220x + 180y at the points O, A, Band C are 0, 3520, 3920 and 3600 respectively.

∴ Maximum profit = ₹ 3920, when 8 electronic and 12 manually operated sewing machines are purchased and sold.

33. Let X be a random variable denoting the total number of green balls drawn in three draws without replacement. Clearly, there may be all green, 2 green, 1 green or no green at all. Therefore, X can take values 0, 1, 2, and 3. Let G_i denote the event of getting a green ball in i^{th} draw.

Now, we have,

P (X = 0) = Probability of getting no green ball in three draws

⇒ P (X = 0) =$P(\overline{G_1} \cap \overline{G_2} \cap \overline{G_3}) = P(\overline{G_1})P(\overline{G_2}/\overline{G_1})P(\overline{G_3}/\overline{G_1} \cap \overline{G_2}) = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{5}{28}$

F (X = 1) = Probability of getting one green ball in three draws

⇒ P (X = 1) = $P\left(\left(G_1 \cap \overline{G_2} \cap \overline{G_3}\right) \cup \left(\overline{G_1} \cap G_2 \cap \overline{G_3}\right) \cup \left(\overline{G_1} \cap \overline{G_2} \cap G_3\right)\right)$

⇒ P (X = 1) = $P\left(G_1 \cap \overline{G_2} \cap \overline{G_3}\right) + P\left(\overline{G_1} \cap G_2 \cap \overline{G_3}\right) + P\left(\overline{G_1} \cap \overline{G_2} \cap G_3\right) + P(\overline{G_1})P(\overline{G_2}/\overline{G_1})P\left(G_3/\overline{G_1} \cap \overline{G_2}\right)$

⇒ P (X = 1) = $\frac{3}{8} \times \frac{5}{7} \times \frac{4}{6} + \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} + \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{15}{28}$

P (X = 2) = P$\left(\left(G_1 \cap G_2 \cap \overline{G_3}\right) \cap \left(\overline{G_1} \cap G_2 \cap G_3\right) \cup \left(G_1 \cap \overline{G_2} \cap G_3\right)\right)$

⇒ P (X = 2) = P$(G_1)\, P\,(G_2/G_1)\, P\left(\overline{G_3}/G_1 \cap G_2\right) + P(\overline{G_1})P\left(G_2/\overline{G_1}\right) P\left(G_3/\overline{G_1} \cap G_2\right)$

$+P\,(G_1)\,P\left(\overline{G_2}/G_1\right) P\left(G_3/G_1 \cap \overline{G_2}\right)$

⇒ P (X = 2) = $\frac{3}{8} \times \frac{2}{7} \times \frac{5}{6} + \frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} + \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} = \frac{15}{56}$

and,

P (X = 3) = P$(G_1 \cap G_2 \cap G_3) = P\,(G_1)\,P\left(\frac{G_2}{G_1}\right) P\left(\frac{G_3}{G_1 \cap G_2}\right) = \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} = \frac{1}{56}$

Therefore, the probability distribution of the number of green balls is given by

X	0	1	2	3
P(X)	$\frac{5}{28}$	$\frac{15}{28}$	$\frac{15}{56}$	$\frac{1}{56}$

OR

The number of ways of choosing two integers (without replacement) from the first six positive integers = ${}^6C_2$ = 15, so the sample space S has 15 equally likely outcomes. These outcomes are:

1, 2; 1, 3; 1, 4; 1, 5; 1, 6; 2, 3; 2, 4; 2, 5; 2, 6; 3, 4; 3, 5; 3, 6; 4, 5; 4, 6; 5, 6

As the random variable X denotes the larger of the two numbers, X can take values 2, 3, 4, 5, 6

(Because 1 is not larger than any number from 1 to 6)

Note that in the sample space S, we have

Larger of two numbers	**Number of outcomes**
2	1
3	2
4	3
5	4
6	5

P(X = 2) = $\frac{1}{15}$, P(X = 3) = $\frac{2}{15}$, P(X = 4) = $\frac{3}{15}$, P(X = 5) = $\frac{4}{15}$, P(X = 6) = $\frac{5}{15}$

∴ The probability distribution of X is:

X	2	3	4	5	6
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

∴ Mean = $\mu = \sum p_i x_i = \frac{1}{15}$ (1 × 2 + 2 × 3 + 3 × 4 + 4 × 5 + 5 × 6)

= $\frac{1}{15}$ (2 + 6 + 12 + 20 + 30) = $\frac{70}{15} = \frac{14}{3}$

Now $\sum p_i x_i^2 = \frac{1}{15}$ $(1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + 4 \times 5^2 + 5 \times 6^2)$

= $\frac{1}{15}$ (4 + 18 + 48 + 100 + 180) = $\frac{350}{15} = \frac{70}{3}$

∴ Variance = $\sum p_i x_i^2 - \mu^2 = \frac{70}{3} - \left(\frac{14}{3}\right)^2 = \frac{70}{3} - \frac{196}{9} = \frac{14}{9}$

34. Quantity of milk in mixture P $= \frac{5}{8}$ part,

quantity of milk in mixture Q $= \frac{13}{16}$ part

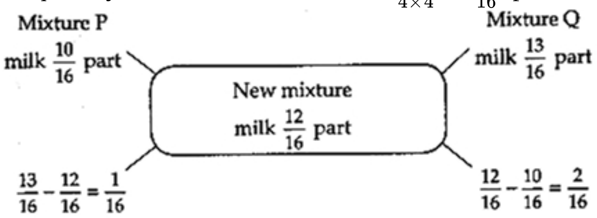
and quantity of milk in new mixture $= \frac{3}{4}$ part

LCM of 8,16 and 4 = 16.

So, qyantity of milk in mixture P = $\frac{5\times2}{8\times2} = \frac{10}{16}$ part,

quantity of milk in mixture Q $= \frac{13}{16}$ part

an quantity of milk in new mixture = $\frac{3\times4}{4\times4} = \frac{12}{16}$ part



$\therefore \frac{\text{Quantity of mixture P}}{\text{Quantity of mixture Q}} = \frac{\frac{1}{16}}{\frac{2}{16}} = \frac{1}{2} = 1:2$

35. Given, P = ₹ 1500000, i = $\frac{12}{12\times100} = \frac{1}{100}$ = 0.01

and n = 8 $\times$ 12 = 96

i. EMI = $\frac{1500000\times0.01\times(1.01)^{96}}{(1.01)^{96}-1}$

$= \frac{1500000\times0.01\times2.5993}{2.5993-1}$

$= \frac{1500000\times0.01\times2.5993}{1.5993}$

= ₹ 24,379.10

ii. Principal outstanding at the beginning of 40th month

$= \frac{EMI\left[(1+i)^{96-40+1}-1\right]}{i(1+i)^{96-40+1}}$

$= \frac{24379.10\times\left[(1.01)^{57}-1\right]}{0.01(1.01)^{57}}$

$= \frac{24379.10\times(1.7633-1)}{0.01\times1.7633}$

$= \frac{24379.10\times0.7633}{0.017633}$

= ₹ 1,055,326.20

iii. Interest paid in 40th payment

$= \frac{EMI\left[(1+i)^{96-40+1}-1\right]}{(1+i)^{96-40+1}}$

$= \frac{24379.10\left[(1.01)^{57}-1\right]}{(1.01)^{57}}$

$= \frac{24379.10\times0.7633}{1.7633}$

= ₹ 10553.26

Section E

36. i. x = 100 - P $\Rightarrow$ P = 100 - x.

R(x) = px = (100 - x) x = 100x - x^2.

ii. P(x) = R(x) - C(x) = (100x - x^2) - $\left(\frac{x^3}{3} - 7x^2 + 111x + 50\right)$

$= -\frac{x^3}{3}$ + 6x^2 - 11x - 50

iii. $\frac{dp}{dx}$ = -x^2 + 12x -11, $\frac{d^2p}{dx^2}$ = -2x + 12

$\frac{dp}{dx}$ = 0 $\Rightarrow$ -x^2 + 12x - 11= 0 $\Rightarrow$ x^2 - 12x + 11 = 0 $\Rightarrow$ x = 1, 11

At x = 11, $\frac{d^2p}{dx^2}$ = 2 $\times$ 11 + 12 = -10 < 0

$\therefore$ Profit is maximum when x = 11

OR

MR = (R(x)) = (100x - x^2) = 100 - 2x

At x = 10, MR = 100 - 2 10 = 80.

37. i. Cost of new machine = ₹ 300000

Scrap value of old machine = ₹ 30000

Hence, the money required for new machine after 7 years

= ₹ 300000 - ₹ 30000 = ₹ 270000

ii. A = ₹ 270000, i = $\frac{5}{100}$ = 0.05, n = 7

iii. A = $R\left[\frac{(1+i)^n-1}{i}\right]$

OR

Cost of new machine = ₹300000

Scrap value of old machine = ₹30000

Hence, the money required for new machine after 7 years

= ₹300000 - ₹30000 = ₹270000

So, we have A = ₹270000, i = $\frac{5}{100}$ = 0.05, n = 7

Using formula, A = $R\left[\frac{(1+i)^n-1}{i}\right]$, we get

270000 = R $\left[\frac{(1.05)^7-1}{0.05}\right]$

[Let x = $(1.05)^7$

$\Rightarrow$ log x = 7 log 1.05 = 7 $\times$ 0.0212 = 0.1484

$\Rightarrow$ x = antilog 0.1484

$\Rightarrow$ x = 1.407

$\Rightarrow R = \frac{270000\times0.05}{(1.05)^7-1}$

$\Rightarrow R = \frac{13500}{1.407-1} = \frac{13500}{0.407}$

$\Rightarrow$ R = 33169.53

Hence, the company should deposit ₹33169.53 at the end of each year for 7 years.

38. i. As per given information,

(x - 8)(y + 10) = xy $\Rightarrow$ 5x - 4y = 80

and x + 16)(y - 10) = xy $\Rightarrow$ 5x + 8y = 80

The matrix equation is $\begin{pmatrix} 5 & -4 \\ -5 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 40 \\ 80 \end{pmatrix}$

ii. adj A = $\begin{pmatrix} 8 & 4 \\ 5 & 5 \end{pmatrix}$

iii. Given system of equations can be written as AX = B

where A = $\begin{pmatrix} 5 & -4 \\ -5 & 8 \end{pmatrix}$, X = $\begin{pmatrix} x \\ y \end{pmatrix}$ and B = $\begin{pmatrix} 40 \\ 80 \end{pmatrix}$

$\Rightarrow$ X = A^{-1} B $\Rightarrow$ X = $\frac{1}{|A|}$(adj A)·B

$\Rightarrow$ X = $\frac{1}{20}\begin{pmatrix} 8 & 4 \\ 5 & 5 \end{pmatrix}\begin{pmatrix} 40 \\ 80 \end{pmatrix} = \frac{1}{20}\begin{pmatrix} 640 \\ 600 \end{pmatrix} = \begin{pmatrix} 32 \\ 30 \end{pmatrix}$

$\Rightarrow$ x = 32, y = 30

Number of children = 32,

amount donated by Ishan = ₹(32 $\times$ 30) = ₹ 960

OR

$A^2 = \begin{pmatrix} 8 & 4 \\ 5 & 5 \end{pmatrix}\begin{pmatrix} 8 & 4 \\ 5 & 5 \end{pmatrix} = \begin{pmatrix} 64+20 & 32+20 \\ 40+25 & 20+25 \end{pmatrix} = \begin{pmatrix} 84 & 52 \\ 65 & 45 \end{pmatrix}$

A^2 - kA + 20I = O

$\Rightarrow \begin{pmatrix} 84 & 52 \\ 65 & 45 \end{pmatrix} - \begin{pmatrix} 8k & 4k \\ 5k & 5k \end{pmatrix} + \begin{pmatrix} 20 & 0 \\ 0 & 20 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 104-8k & 52-4k \\ 65-5k & 65-5k \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow$ 65 - 5k = 0 $\Rightarrow$ k = 13